TIME-DEPENDENT ACCRETION α–DISKS IN BINARY SYSTEMS

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ABSTRACT

We review the properties of analytical solutions to the basic equation of time-dependent accretion. A new analytical solution describing a time-dependent disk in a binary system at outburst which has a fixed tidally-truncated outer radius is applied to 1975 X-ray nova burst of A0620-00. The X-ray flux of multicolor (black-body) α-disk decays quasi-exponentially with time. Modeling of the light curves of A0620-00 before and after second maximum is presented.

Key words: accretion disks; close binaries; X-ray novae; A0620-00 (=V616 Mon).

1. VISCOUS DISK EVOLUTION: PROBLEM AND ANALYTICAL SOLUTIONS

We consider an accretion disk around a gravitating object of mass M: (1) a disk is geometrically thin in a sense that its half-height at a given radius is much less than this radius: $z_o << r$; (2) the time scale, over which the radial disk structure changes ('the viscous time scale'), is much longer than the hydrostatic timescale, over which the vertical structure varies, and than the thermal timescale, over which the disk temperature changes: $\tau_{\rm vis} >> \tau_{\rm hyd} >> \tau_{\rm th}$.

From the Euler equation on φ and the continuity equation, integrated along the height of a disk, one can derive the so-called viscous diffusion equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{2}\,\frac{(GM)^2}{h^3}\,\frac{\partial}{\partial h}\left(\left[\frac{\partial h_*}{\partial h}\right]^{-1}\frac{\partial F}{\partial h}\right)\,, \qquad (1)$$

where Σ is the surface density of a disk; h_* is the specific angular momentum of a disk in a general case; h is the Keplerian angular momentum;

$F = W_{r\varphi}\, r^2$ has the meaning of the total moment of viscous forces acting between adjacent layers, divided by 2π. $W_{r\varphi}$ is the height-integrated shear stresses between adjacent layers. In an α–disk (Shakura & Sunyaev, 1973) $W_{r\varphi} = \alpha\,\Sigma\,v^2_{sound}$.

In a general case the particles in a disk are subjected to the gradient of pressure, which is neglected in a standard disk model when particles move along Kepler orbits.

Hereafter, we assume that the disk is Keplerian. For a Keplerian disk, Eq. (1) can be re-written:

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{4}\,\frac{(G\,M)^2}{h^3}\,\frac{\partial^2(\Sigma\,\nu\,h)}{\partial h^2}\,, \qquad (2)$$

where ν is the kinematic coefficient of turbulent viscosity.

If F is linear on the surface density, in other words, ν is a function of radius alone, Eq. (2) becomes a linear differential equation of diffusion type. Various radial dependences of ν were considered by Weizsäcker (1948), Lüst (1952), Lynden-Bell & Pringle (1974). Note that the viscous timescale, which is $\sim r^2/\nu$, is temporally constant in such a consideration. Solutions can be presented as the superposition of particular solutions.

A more general case of non-linear Eq. (2) admits of self-similar solutions when ν is a power–law function of Σ and r (Filipov, 1984; 1988; Lyubarskii & Shakura, 1987). Lyubarskii & Shakura (1987) used the α–model of disk structure to find a relation between F and Σ (see Eq. (6) below). The self-similar solutions can be presented in the form:

$$F(h,t) = f_1(t) \times f_2(h/H(t))\,, \qquad (3)$$

where f_1, f_2, and H are functions of single variables.

In our work (Lipunova & Shakura, 2000) we investigated Eq. (2) in a case of a standard α–disk rotating around a component of a binary system. We used the relation between ν and Σ from Lyubarskii & Shakura (1987) and found an asymptotic solution that describes a decay of accretion. The solution has the form

$$F(h,t) = F(t) \times f(h/h_o)\,. \qquad (4)$$

The quantity $h_o = (G\,M\,r_{\rm out})^{1/2}$ corresponds to the outer boundary of a truncated disk.

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2. OUTER BOUNDARY CONDITIONS

We propose that the mass transfer rate between the components of a binary system is negligible in comparison with the accretion rate in the disk. In such a situation the accretion rate in the disk decreases eventually.

One of the solutions of Lyubarskii & Shakura (1987) deals with a decay of accretion onto a centre, but it cannot be applied to the disk under question because their solution corresponds to an infinitely spreading structure.

In a binary system the spreading of the disk is to be confined by the tidal interactions. The tidal torque produced by a secondary star has strong radial dependence (Papaloizou & Pringle, 1977). As Ichikawa & Osaki (1994) showed, the tidal effects are generally small in the accretion disk, except near to the tidal truncation radius, which is given by the last non-intersecting periodic particle orbit in the disk (Paczyński, 1977). They concluded that once the disk expands to the tidal truncation radius, the tidal torques prevent the disk from expanding beyond the tidal radius.

We consider the size of the disk to be invariant over a certain period of an outburst. As the drain of angular momentum occurs in a narrow region near the truncation radius, we treat the region near this radius as the δ-type channel, not considering the details of the process. In other words, the smooth behaviour of spatial factor f in Eq. (4) (which increases as $\propto r^{1/2}$ in the inner parts of the disk, then flattens, reaches the maximum and drops down near $r_{\rm out}$ due to tidal torque) is analytically treated as increasing, flattening, and reaching maximum at $r_{\rm out}$, which is the end of the disk (this profile is shown in Fig. 1). Thus we propose the boundary conditions as follows:

$$f(1) = 1, \quad f'(1) = 0 \ . \tag{5}$$

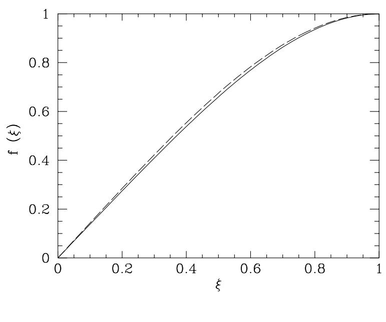

Figure 1. The radial profile of $F(h,t)$, $f(\xi)$, in two cases: Thomson opacity (solid line) and free-free opacity (dashed line). $\xi = h/h_{\rm o}$.

3. VISCOUS EVOLUTION OF A STANDARD DISK IN A BINARY SYSTEM

A relation between Σ, F, and h is provided by the following expression:

$$\Sigma = \frac{(GM)^2\, F^{1-m}}{2\,(1-m)\, D\, h^{3-n}} \tag{6}$$

[Filipov (1984), Lyubarskii & Shakura (1987)], where D, defined by the specific vertical structure, is a function of α, opacity coefficient, and dimensionless values, which are combinations of the characteristic physical parameters of the disk. The value of D can be derived from the consideration of the disk vertical structure using the results of the work by Ketsaris & Shakura (1998). $m = 2/5$, $n = 6/5$ if the Thomson scattering dominates the opacity in the accretion disk, and $m = 3/10$, $n = 4/5$ if the free-free and free-bound transitions do.

Substituting expressions (4) and (6) in Eq. (2), using the conditions (5), one finds the asymptotic law of time variations:

$$F(t) = \left(\frac{h_{\rm o}^{n+2}}{-\lambda\, m\, D\,(t+t_0)}\right)^{1/m} = F_{\rm o}\, \left(1+\frac{t}{t_{\rm o}}\right)^{-1/m} , \tag{7}$$

λ and $t_{\rm o}$ are integration constants, and $f(h/h_{\rm o})$ is displayed in Fig. 1.

The accretion rate in the disk is as follows:

$$\dot M(h,t) = -2\pi f'(h/h_{\rm o}) F(t)/h_{\rm o} \ , \tag{8}$$

and the bolometric luminosity

$$L(t) = \eta\, \dot M(t)\, c^2 \tag{9}$$

$$\propto (t + t_{\rm o}^{\rm Th})^{-5/2} \qquad \text{Thomson opacity}$$

$$\propto (t + t_{\rm o}^{\rm ff})^{-10/3} \qquad \text{free-free opacity}$$

The model assumes two separate regimes, which is not quite true. Indeed, for a long time the inner part of the disk should be scattering-dominated, the lower the accretion rate, the smaller this part. We note that $\dot M$ decay rate is regulated by the outer parts of the disk. Obtaining an exact solution needs a consideration of combined free-free and Thomson opacity of the gas, but the smoothness of the transition between the opacity regimes (Fig. 2) supports the reliability of the obtained solution.

There is a value of t which corresponds to the Eddington limit $L_{\rm Edd} \approx 1.3 \times 10^{38}\, m_{\rm x}$ erg s^{-1}. This means that the real source evolution could be described in our model only at later t. Thus, generally speaking, the solution before this time moment appears inapplicable. As seen in Fig. 2, the applicable part of the solution belongs almost entirely to the free-free opacity regime.

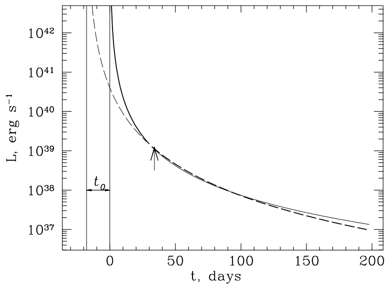

Figure 2. Bolometric luminosity in the two opacity regimes calculated for parameters: $m_{\rm x} = 3$, $\alpha = 0.3$, $\mu = 0.5$, $r_{\rm out} = R_\odot$. Small arrow marks the transition from the Thomson opacity regime (solid line) to the free-free regime (dashed line). Here, t is counted from the singularity of the asymptotic solution in Thomson regime.

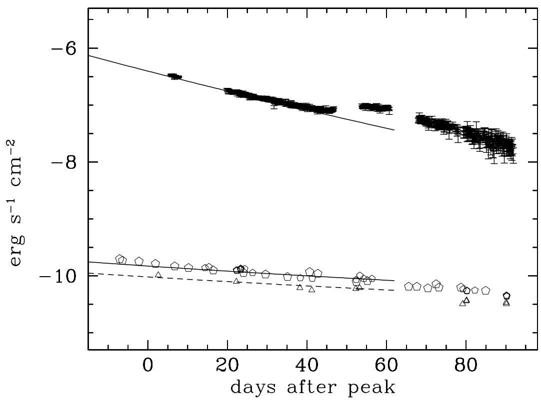

Figure 3. The asymptotic fit for the X-ray nova A0620-00. The Ariel-5 *3–6 keV light curve is fitted by upper solid line. Optical B (solid) and V (dashed) light curves are shown for the case when no irradiation is involved. The model is as follows: $M_{\rm x} = 10M_\odot$, $\alpha = 0.35$, $M_{\rm opt} = 0.5M_\odot$, $\mu = 0.5$, $r_{\rm out} = 2.15R_\odot$, $d = 0.93$ kpc, E(B− V)=0.39, black-body local spectrum, free-free opacity, Paczyński & Wiita potential. The bolometric luminosity of the disk at peak is $\approx 0.5L_{\rm Edd}$; the effective temperature of the outer disk at $t \approx 50$ days is ~ 9200 K and in the central plane is ~ 22400 K.*

We remark that the observed X-ray light curves can have *different* (most probably, steeper) law of decay. Indeed, the energy band of an X-ray detector usually covers the region harder 1 keV where the multicolor photon spectrum of the disk (having appropriate temperature) can have turnover from $-2/3$ power law into exponential fall. This turnover is expected to change its position due to variations in temperature of the disk. Thus the law of variation of X-ray flux is roughly proportional to $\exp\left(-h_{\rm Pl}\,\nu / k\,T^{\rm eff}(t)\right)$. In the free-free regime of opacity we have $T^{\rm eff}(t) \propto L^{1/4}(t) \propto t^{-10/12}$. Consequently, the observed X-ray flux varies like $\exp\left(-t^{5/6}\right)$, which is quite close to exponential behaviour.

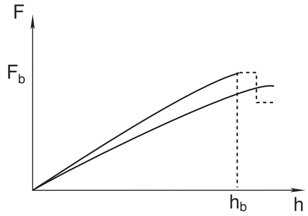

Figure 4. A schematic picture of a model explaining the second maximum.

4. OBSERVATIONAL APPEARANCE

There are several requirements to a source evolution of which is to be described by the presented model of a time-dependent α–disk: (1) low mass rate to the outer boundary of a disk in comparison with the accretion rate in a disk; (2) if the matter supply of a disk is steady, the agglomeration of matter must occur that triggers a subsequent outburst; (3) the accretion rate during a burst is not too low for the disk to be fully ionized; (4) the accretion rate during a considered time interval of a burst is not too large to avoid a supercritical regime when $P_{\rm rad} > P_{\rm gas}$ in a substantial part of a disk, and a disk is not geometrically thin [$z_0/h \sim 1/2$ (Wang & Zhou, 1999; Lipunova, 1999)].

We propose that such an accretion disk is present in an X-ray nova during an outburst when it is in the *high/soft* state. In this state, X-ray novae are established to emit a black-body multicolor disk spectrum, and a corona component is only additional at higher X-rays.

We model the 1975 burst of A0620-00, using the X-data in 3–6 keV band of *Ariel-5* (Kaluzienski et al., 1977) and the data in B and V optical bands (Lloyd et al., 1977; Kuulkers, 1999). An example of fitting is displayed in Fig. 3. We used the Paczyński & Wiita pseudo–Newtonian potential to describe a gravitational potential of a black hole.

Modeling the X-ray and the optical light curves of A0620-00, we can estimate α and obtain 'distance–mass of the compact object' relation (Lipunova & Shakura, 2000b).

5. SECOND MAXIMUM

We propose a simple model of the inward progress of a convective zone. At ~ 50 days the convective zone starts from the outer radius where T_c is calculated to be 22000 K and $T_{eff} \sim 9000$ K. In the convective zone the turbulent motion strengthens and α arises so that an accretion rate increases (Fig. 4). Behind the convective zone the gas is weakly ionized, and the accretion rate drops. Thus, emission from the outer part of the disk is negligible. To find a time-dependence of accretion rate we can write:

$$-\frac{\mathrm{d}M_*}{\mathrm{d}t} = \dot{M}_{\rm in} + \Delta_{\rm t} M_{\rm b} \ , \qquad (10)$$

where $\dot{M}_{\rm in}$ is the accretion rate through the inner boundary, M_* is a mass of the ionized part of the disk, $\Delta_{\rm t} M_{\rm b}$ is a mass decrease per unit time due to the inward motion of the convective zone. We take $F_{\rm b} = \alpha_2\, \Sigma_{\rm b}\, v_{\rm b}^2\, R_{\rm b}$, where α_2 is an increased value of turbulent parameter, and $v_{\rm b}$ is a fixed sound velocity corresponding to the temperature of the convective zone. Using Eq. (6), we obtain:

$$\Sigma_{\rm b} \propto R_{\rm b} \, . \qquad (11)$$

Lower index 'b' corresponds to the values at the boundary of the convective zone. The mass of the ionized part of the disk $\propto \Sigma_{\rm b}\, R_{\rm b}^2 \propto R_{\rm b}^3$. Subsequently,

$$\Delta_{\rm t} M_{\rm b} = \Sigma_{\rm b}\, 2\pi\, R_{\rm b}\, \dot{R}_{\rm b} \propto \frac{\mathrm{d}M_*}{\mathrm{d}t} \, . \qquad (12)$$

Assuming $\dot{M}_{\rm in} \approx -2\,\pi\, F_{\rm b}/h_{\rm b}$ (c.f. Eq. (8)) we have

$$\dot{M}_{\rm in} \propto R_{\rm b}^{5/2} \propto M_*^{5/6} \, . \qquad (13)$$

Thus, Eq. (10) becomes the following:

$$-\frac{\mathrm{d}M_*}{\mathrm{d}t} = A\, M_*^{5/6} + B\, \frac{\mathrm{d}M_*}{\mathrm{d}t} \, . \qquad (14)$$

One readily obtains the solution

$$R_{\rm b} \propto (t-t_0)^2 \ , \qquad \dot{M}_{\rm in} \propto (t-t_0)^5 \ ,$$

where a constant t_0 is to be obtained from initial conditions at $t = 0$ when convection starts from the outer boundary of the disk. The example of fitting is shown in Fig. 5.

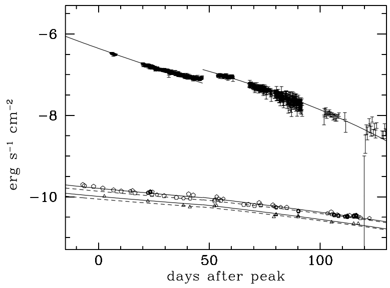

Figure 5. B and V light curves are fitted by solid (irradiation of disk involved) and dashed lines (no irradiation). The model is the same as in Fig. 3, except $\alpha = 0.37$, $\alpha_2 = 0.45$, $d = 1$ kpc. The right parts of the curves represent modeling after the second maximum.

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REFERENCES

Barret D., McClintock J.E., Grindlay J.E., 1996, ApJ 473, 963

Chen W., Shrader C.R., Livio M., 1997, ApJ 491, 312

Filipov L.G., 1984, Adv. Space Res. 3, 305

Filipov L.G., 1988, Adv. Space Res. 8, 141

Ichikawa S., Osaki Y., 1994, PASJ 46, 621

Kaluzienski L.J., Holt S.S., Boldt E.A., Serlemitsos P.J., 1977, ApJ 212, 203

Ketsaris N.A., Shakura N.I., 1998, Astronomical and Astrophysical Transactions 15, 193

Kuulkers E., 1999, New Astronomy Rev. 42, 1

Lipunova G.V., 1999, Astronomy Letters 25, 508

Lipunova G.V., Shakura N.I., 2000, A&A 356, 363

Lipunova G.V., Shakura N.I., 2000b, In: Third Microquasar Workshop Proceedings, Granada, *in press*

Lloyd C., Noble R., Penston M.V., 1977, MNRAS 179, 675

Lüst R., 1952, Naturforschg 7a, 87

Lynden-Bell D., Pringle J.E., 1974, MNRAS 168, 603

Lyubarskii Yu.E., Shakura N.I., 1987, SvA 13, 386

Paczyński B., 1977, ApJ 216, 822

Papaloizou J., Pringle J.E., 1977, MNRAS 181, 441

Robertson B.C.S., Warren P.R., Bywater R.A., 1976, Information Bulletin on Variable Stars, 1173, 1

Shakura N.I., Sunyaev R.A., 1973, A&A 24, 337

Tanaka Y., 1992, In: Makino F., Nagase F. (eds.) Ginga Memorial Symp., ISAS, 19

Wang J.-M., Zhou Y.-Y., 1999, ApJ 516, 420

Weizsäcker C.F., 1948, Z. Naturforschg 3a, 524

Wu C.-C., 1976, A&A 50, 445